Randomization Tests of Causal Effects Under General Interference

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Medellín, Colombia

Medellín, Colombia

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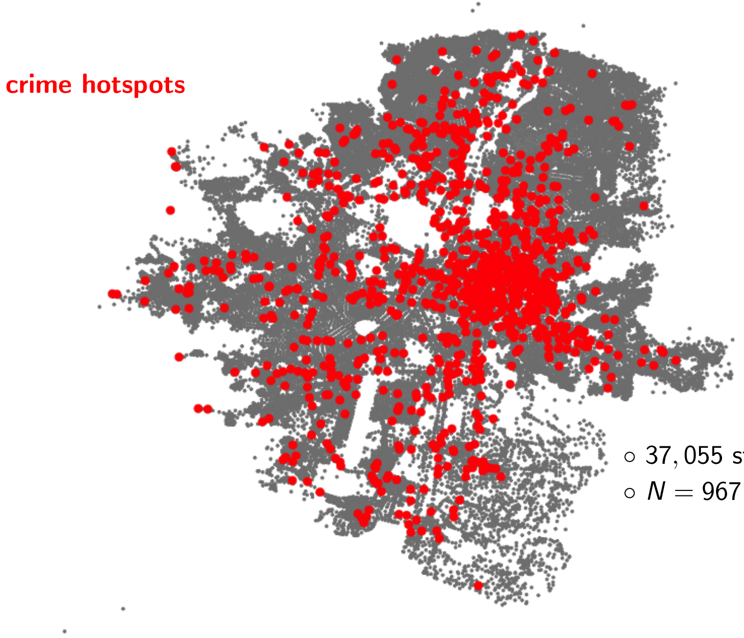
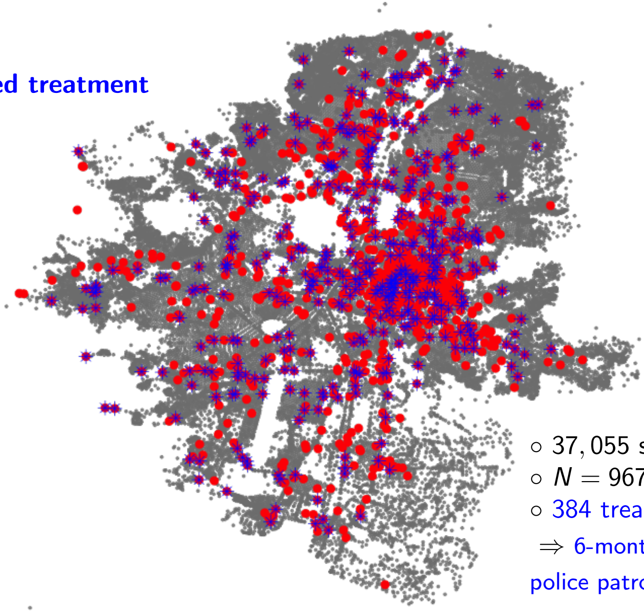


- $\circ$ 37,055 street segments
- $\circ$ $N = 967$ "hotspots".

Medellín, Colombia



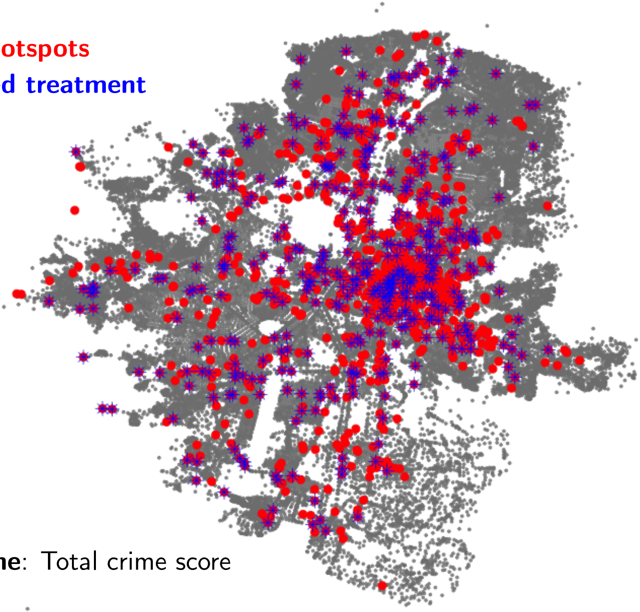
- 37,055 street segments
- $N = 967$ "hotspots".
- 384 treated at random.

$\Rightarrow$ 6-month increase in police patrolling time

Medellín, Colombia

crime hotspots

observed treatment



Outcome: Total crime score

Questions

How does the intervention affect crime?

$\rightarrow$ direct effect?

$\rightarrow$ spillovers to adjacent streets?

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$\rightarrow$ direct effect?

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We will focus on testing for **spillovers**.

We prefer a model-free approach, so we will use the randomization method of inference.

Notation and data

N units (streets) indexed by $i = 1, 2, \ldots, N$.

Denote:

$Z = (Z_1, \ldots, Z_N)$ as binary treatment; $P(Z) =$ design;
$Y = (Y_1, \ldots, Y_N)$ as vector of observed outcomes.
$\hookrightarrow$ *will also use Z^{obs}, Y^{obs} to emphasize observed quantities; and Z', Y' will denote "counterfactuals".*

Hotspots received increased policing, while non-hotspots lost about $\sim$5min of patrol time.

Outcome is a weighted average of crime indicators:
$\hookrightarrow$ *0.550 for homicides, 0.112 for assaults, 0.221 for car and motorbike theft, and 0.116 for personal robbery (Collazos et al., 2019).*

Outline

1. No interference: classical Fisher Randomization Test (FRT).

2. Interference
   - Treatment exposures.
   - The null-exposure graph.
   - "Clique-based" FRT for spillovers.
   - Related work.

3. Application in Medellin experiment.

4. (extra, if time) Power study; extensions.

Classical approach: no interference

Assume no interference: Outcome of unit depends only on its own treatment assignment.

↪ *Only two potential outcomes,* $Y_i(0), Y_i(1)$*, for every* i*.*

Unrealistic in this application. But helps build the intuition.

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Classical question of randomization inference:
Does treatment have an effect *at all*?

$$\mathbf{H_0}: \quad Y_i(0) = Y_i(1) \text{ for every } i.$$

Key implication of H_0 is that Y is fixed across all possible randomizations.

Fisher randomization test (1935)

$\mathbf{H_0}$: $\quad Y_i(0) = Y_i(1),$ for every i.

The procedure:

Choose test statistic $T = t(y, z)$ (e.g., difference in means).

1. $T^{\text{obs}} = t(Y^{\text{obs}}, Z^{\text{obs}})$.
2. Sample $Z' \sim P(Z')$, store $T_R = t(Y^{\text{obs}}, Z')$.
3. p-value $= \mathbb{E}\left[\mathbb{1}\{T_R \geq T^{\text{obs}}\}\right]$.

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Proof of validity:

$$t(Y^{\text{obs}}, Z') \overset{H_0}{=} t(Y', Z') \overset{d}{=} t(Y^{\text{obs}}, Z^{\text{obs}})$$

"$T_R \sim T^{\text{obs}}$ (under null)"

Advantages of Fisherian randomization

- Exact. The test is valid in finite samples.

- Minimal assumptions. No model for Y.

- Robust. Same answer under some transformations of Ys.

Our goal is to use Fisherian randomization under interference.

"No interference" assumption is too strong ...

Assumption of no interference is not realistic in our application.
Spillovers are actually a quantity of interest.

First problem is notational:
In general, $Y_i(z)$ is the potential outcome of i under population treatment assignment z.

$\hookrightarrow$ *More outcomes than just* $Y_i(0), Y_i(1)$.

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In principle there could be 2^N potential outcomes. Impractical.

To make progress we can use the concept of **exposure functions**.

Treatment exposures

For any given Z, unit i is exposed to "something more" than Z_i. We assume unit i's exposure is defined by a function:

$$f_i : \{0,1\}^N \to \mathcal{E}.$$

$\mathcal{E} =$ set of possible exposures (short-range spillover, medium-range spillover, pure control, etc.)

e.g., $f_i(z) = (z_i, \sum_{j \neq i} g_{ij} z_j) \in \{0,1\} \times \{0,1,2,\ldots\}$, where g_{ij} indicates whether i and j can influence each other (classmates or neighbors).

Definition of $\mathcal{E}, \{f_i\}$ depends on the substantive scientific question.

What's in a treatment exposure?

Intuitively, we think that $f_i(z)$ defines "equivalence classes" across the population assignments; e.g.,

$$Y_i(z) = Y_i(z'), \text{ if } f_i(z) = f_i(z').$$

This assumption is not necessary for our method but helps with interpretation of our results.

Alternative: when f_i are unspecified we may consider marginal estimands, e.g., $E\{Y_i(z_i = 1, z_{-i})\} - E\{Y_i(z_i = 0, z_{-i})\}$;

$\hookrightarrow$ *see Aronow and Samii (2019); Savje (2019).*

Question: Is there a short-range spillover effect?

We can use exposures $\{f_i\}$ to study spillovers/interference.

$$\mathbf{H_0}: \; Y_i(z) = Y_i(z') \text{ for every } i, z, z',$$

$$\text{such that } f_i(z), f_i(z') \in \{\texttt{short}, \texttt{control}\}.$$

Here, we defined:

$$f_i(z) = \begin{cases} \texttt{short}, & z_i = 0, \text{dist}_i < 125\text{m} \\ \texttt{control}, & z_i = 0, \text{dist}_i > 500\text{m} \\ \texttt{neither}, & \text{otherwise.} \end{cases}$$

dist_i = distance to closest treated street.

Use classical Fisherian randomization? Not quite ...

Recall, $T_R \sim T^{\text{obs}}$ under H_0 for things to work. However,

$$T_R = t(Y^{\text{obs}}, Z') \overset{\cancel{H_0}}{\neq} t(Y', Z^{'}) \overset{d}{=} t(Y^{\text{obs}}, Z^{\text{obs}}) = T^{\text{obs}}.$$

The null is relevant for only 2 out of the 3 possible exposures:

$$\mathbf{H_0}: \quad Y_i(\texttt{short}) = Y_i(\texttt{control}) \overset{?}{=} Y_i(\texttt{neither}), \text{ for every i.}$$

In this case, the null is **not sharp**. We cannot impute missing potential outcomes $Y^{'}$ freely under any $Z^{'}$.

Testing $Y_i(\texttt{short}) = Y_i(\texttt{control}), \forall i$.

Idea: If we focus on units only exposed to `short` or `control` then we can impute their missing outcomes in the randomization test.

$\hookrightarrow$*i.e., conditional randomization test (Aronow, 2012) (Athey et al, 2018) (Basse et al, 2019). Will discuss later.*

Basse et al (2019) formalize this approach: given Z^{obs} we condition on some event C according to $P(C|Z^{\text{obs}})$, known as the conditioning mechanism. The conditional test is valid as long as:

1. Can impute outcomes conditional on C.
2. We randomize according to the correct conditional distribution:

$$P(Z'|C) \propto \underbrace{P(C|Z')}_{\text{conditioning mech.}} \cdot \underbrace{P(Z')}_{\text{design}}.$$

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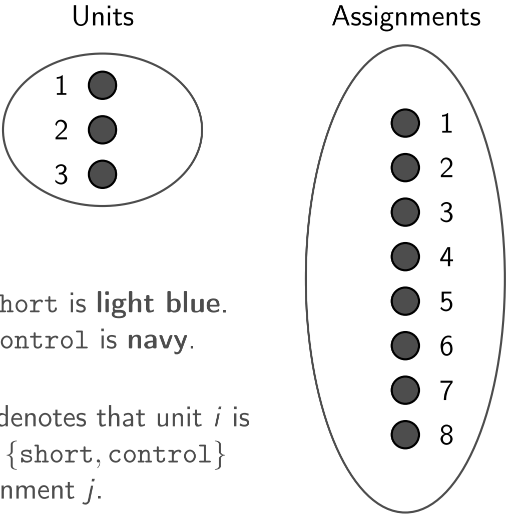
$\star$ But how to construct $P(C|Z^{\text{obs}})$?

The null exposure graph



Exposure `short` is **light blue**.
Exposure `control` is **navy**.

edge (i, j) denotes that unit i is exposed to $\{\texttt{short}, \texttt{control}\}$ under assignment j.

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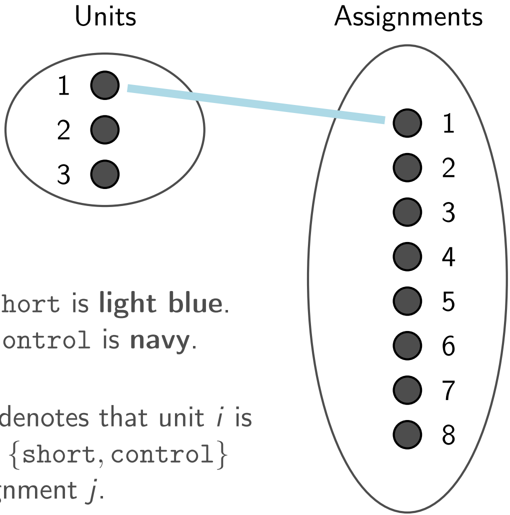

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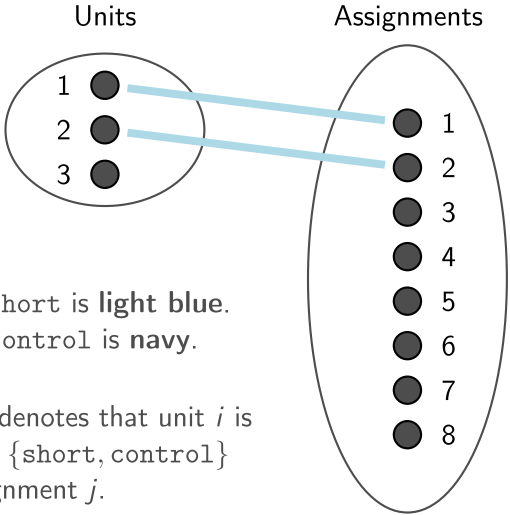

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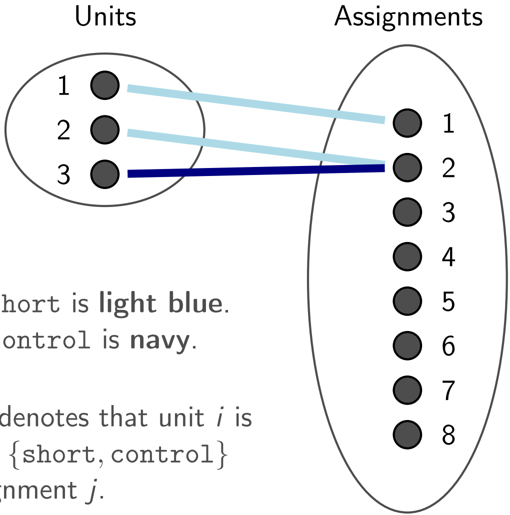

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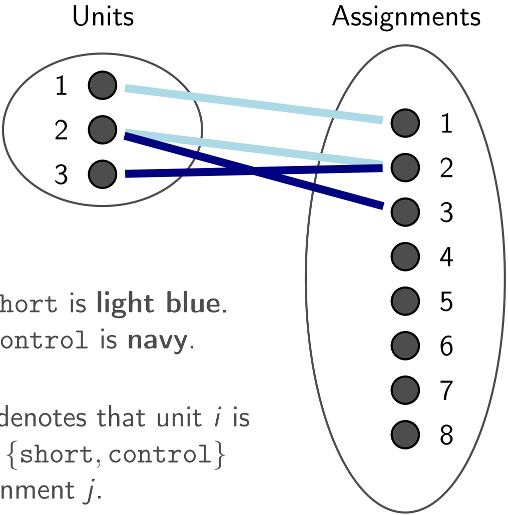

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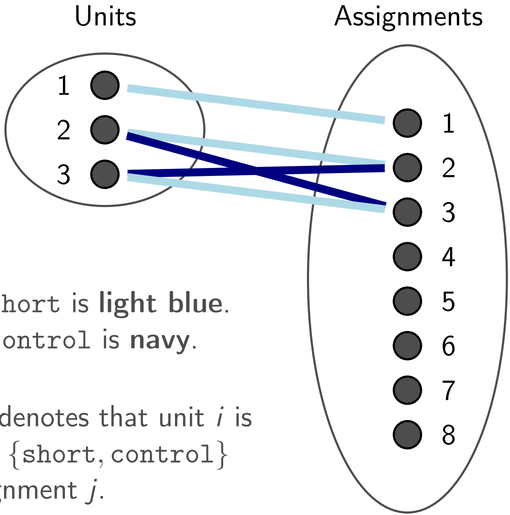

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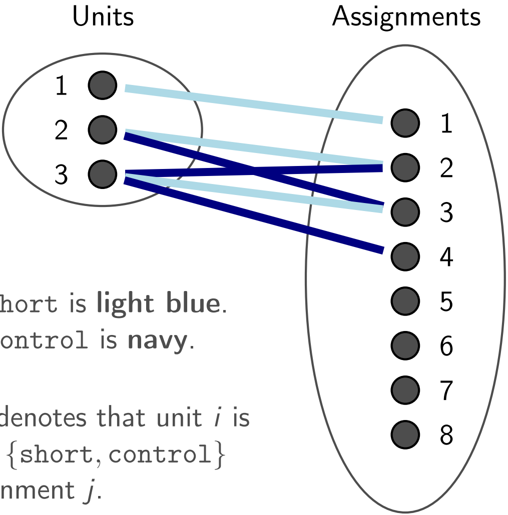

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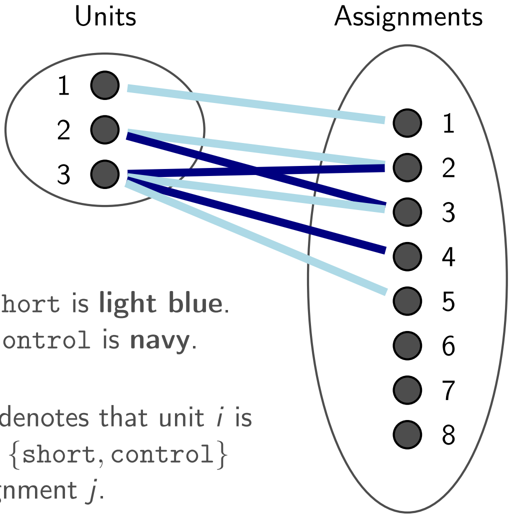

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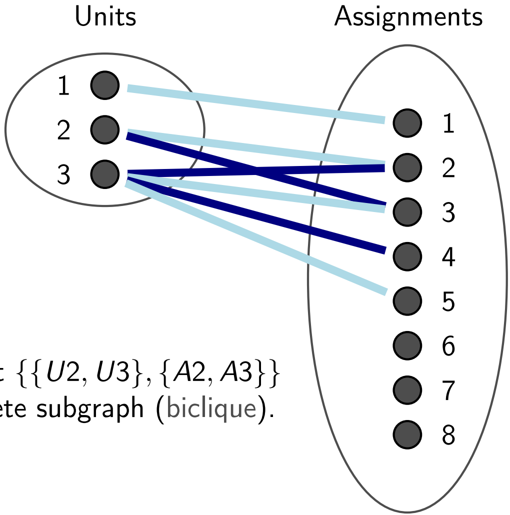
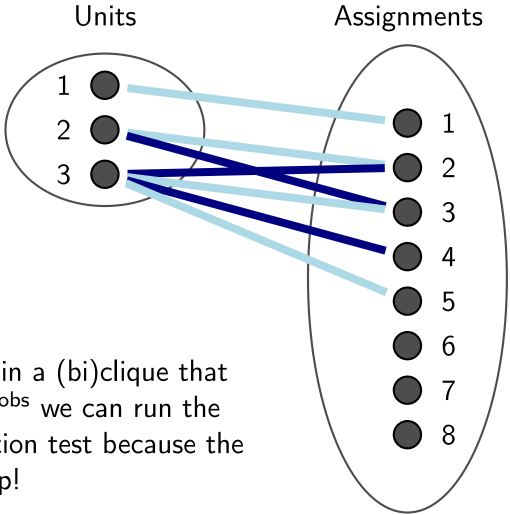


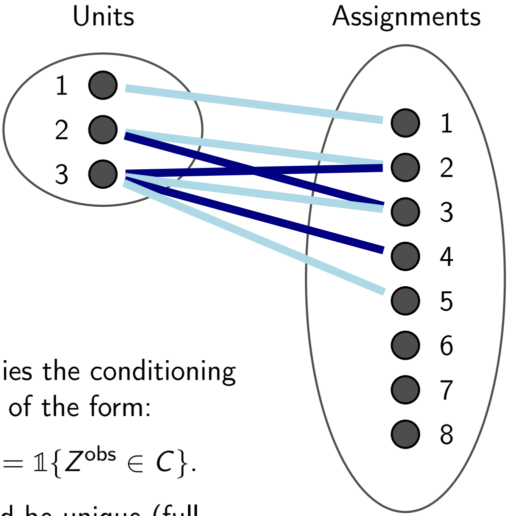
Notice that $\{\{U2, U3\}, \{A2, A3\}\}$ is a complete subgraph (biclique).

The null exposure graph



Idea: Within a (bi)clique that contains Z^{obs} we can run the randomization test because the null is sharp!

The null exposure graph



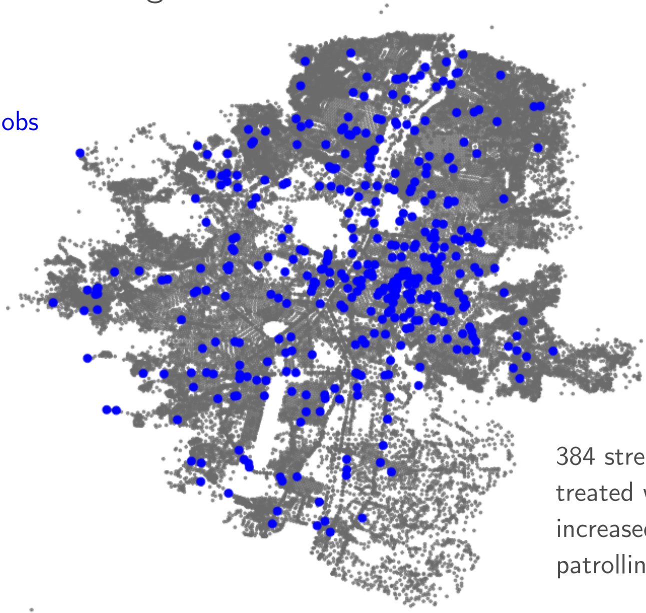
Idea: Implies the conditioning mechanism of the form:

$P(C|Z^{\mathrm{obs}}) = \mathbb{1}\{Z^{\mathrm{obs}} \in C\}$.

So C should be unique (full definition coming soon).

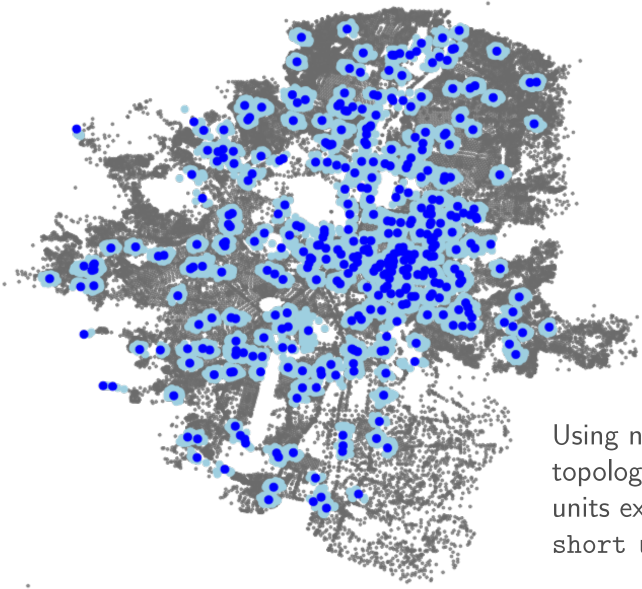
Returning to the map

The observed assignment



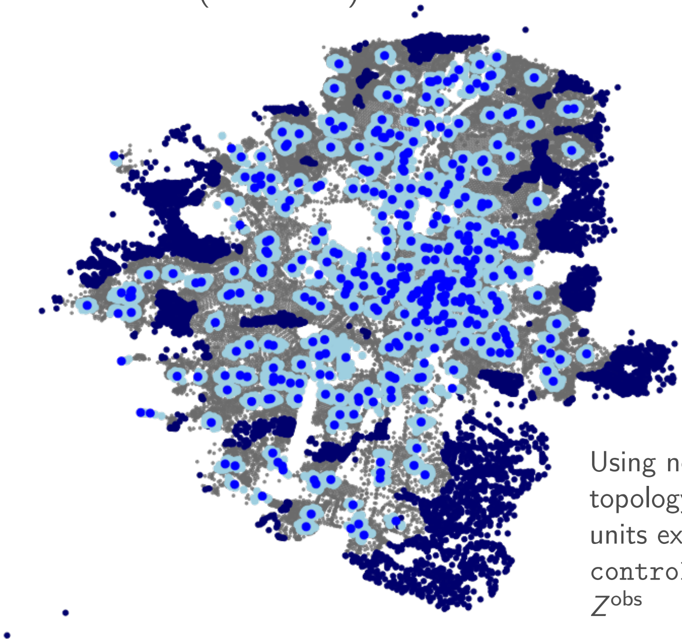
384 streets are treated with increased police patrolling

Short-range spillover units (`short`)



Using network topology, color units exposed to `short` under Z^{obs}

Pure control units (`control`)



Using network topology, color units exposed to `control` under Z^{obs}

We can remake these pictures for every assignment Z drawn from design $P(Z)$ ...

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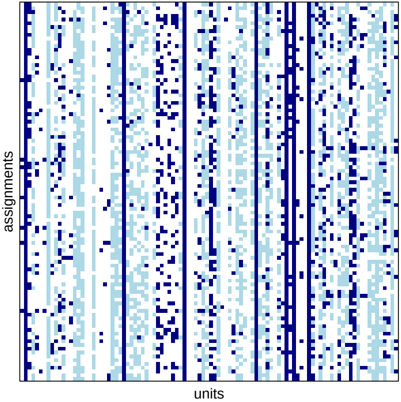
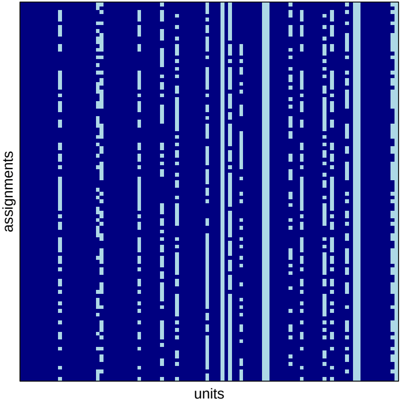
$\rightarrow$ The output is our null exposure graph!

Null exposure graph and clique



null exposure graph



clique (zoomed-in)

Null-exposure graphs: summary

∘ A null-exposure graph, G_f, is thus uniquely defined given $H_0, \{f_i\}$. (see formal defn., Slide 42).

∘ H_0 is sharp in a clique of G_f. So, we run a conditional randomization test within a clique.

↪*i.e.,* $P(C|Z^{obs}) = \mathbb{1}\{Z^{obs} \in C\}$.

∘ Such test requires a "clique test statistic" $t(y, z; C)$ where C is a clique in G_f such that

$$t(y, z; C) = t(y', z'; C), \text{ if } y_C = y'_C \text{ and } z_C = z'_C.$$

↪y_C, z_C *are sub-vectors of* y, z *only with units/assignments in* C.

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⋆ But which clique to condition on?

A naive test (which doesn't work)

Not all approaches lead to a valid test. For example, consider:

1. Given Z^{obs} calculate maximum clique in null-exposure graph, G_f, that contains Z^{obs}, say,

$$C^* = \texttt{mc}(Z^{\text{obs}}; G_f); \qquad (\texttt{mc} = \text{"}\texttt{max clique}\text{"}).$$

2. Condition the randomization test on C^*, resampling assignments according to

$$P_R(Z'|C^*) = \frac{\mathbb{1}\{Z' \in C^*\}P(Z')}{P(C^*)}.$$

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Proof of invalidity:

The correct conditional distribution is:

$$P(Z'|C^*) = \frac{P(C^*|Z')P(Z')}{P(C^*)} = \frac{\mathbb{1}\{\texttt{mc}(Z'; G_f) = C^*\}P(Z')}{P(C^*)} \neq P_R.$$

Clique-based randomization test

1. **Decompose:** Compute biclique decomposition* $\mathcal{C}$ of G_f. Pick out clique containing Z^{obs}, call it C.

2. **Condition:** Compute $T^{\text{obs}} = t(Y^{\text{obs}}, Z^{\text{obs}}; C)$ given C.

3. **Summarize:**
   p-value $= \mathbb{E}\left[\mathbb{1}\{t(Y^{\text{obs}}, Z'; C) \geq T^{\text{obs}}\} \mid C\right]$.
   $\hookrightarrow$ *Here, we sample with respect to*

$$P_R(Z'|C) \propto \mathbb{1}\{Z' \in C\} \cdot \underbrace{P(Z')}_{design}$$

$\star$ (see formal defn., Slide 43).

Why this works

- The randomization distribution in the test is:

$$P_R(Z'|C) = \frac{\mathbb{1}\{Z' \in C\}P(Z')}{P(C)}.$$

- The correct conditional distribution is:

$$P(Z'|C) = \frac{P(C|Z')P(Z')}{P(C)} = \frac{\mathbb{1}\{C \in \mathcal{C}\}\mathbb{1}\{Z' \in C\}P(Z')}{P(C)} = P_R,$$

whenever we use only cliques from decomposition $\mathcal{C}$.

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Proof of validity:

$$t(Y^{\text{obs}}, Z'; C) \overset{H_0,\,C}{=} t(Y', Z'; C) \overset{d}{=} t(Y^{\text{obs}}, Z^{\text{obs}}; C)$$

"$T_R \sim T^{\text{obs}}$ (under null conditional on C)"

Biclique decomposition

- Finding cliques is NP-hard (Peeters, 2003; Zhang et al, 2014).
- We use the "Binary Inclusion-Maximal Biclustering Algorithm", which uses a "divide and conquer" method to find cliques (`Bimax`, Prelic et. al, 2006).

  ↪*works fine for hundred nodes/thousands edges.*
- Our method is **constructive**, still needs to be optimized.

  ↪*i.e., different biclique decompositions will have different power properties, but all are* ***valid****.*

Related work

We can also use our framework to describe related work:

- Aronow (2012) and Athey et al (2018) effectively propose to randomly sample *focal units* on one side, and then find the *maximum induced* clique to condition on.

  $\hookrightarrow$ *General procedure but the random selection of focals does not exploit the problem structure — Loss of power.*
  (See also power study, Slide 44.)

- Basse et al (2019) develop a clique decomposition that provably leads to permutation test under a setting with clustered interference.

  $\hookrightarrow$ *Case-by-case analysis — Cannot generalize.*

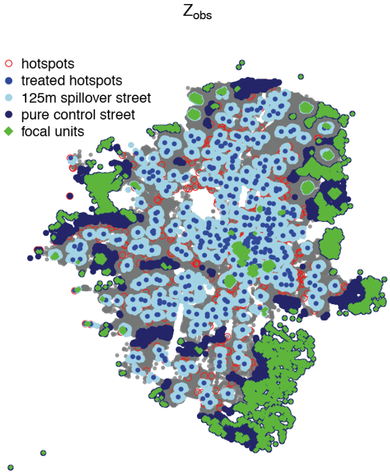
Spatial interference: Medellin data

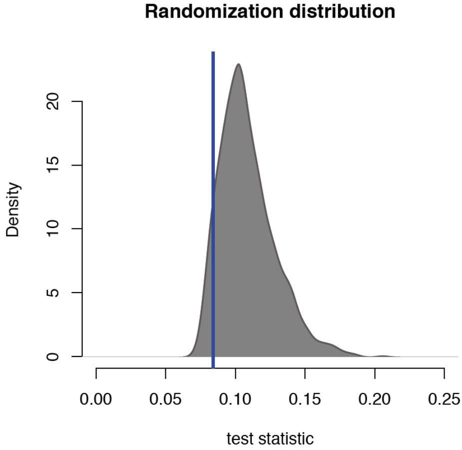
Statistics of the null-exposure graph:

- #units = 37,055.
- #assignments = 10,000.
- #edges = 163,836,445.
- density (#edges / total #of possible edges) = 44.2%

Statistics of the clique we condition on:

- #units in clique = 3,981.
- #assignments in clique $\approx 1,000$.





Focal units (in green) are in downtown and outskirts.
Clique test automatically discovers this pattern.

Varying radius of short-range effect

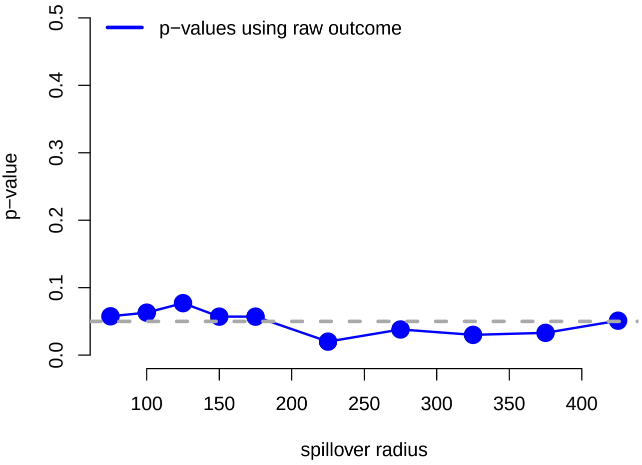


Figure: P-values for clique tests with varying spillover radius.

Concluding thoughts

- New method is presented for testing causal effects under general interference using null exposure graphs and bicliques.
- Structure is placed on null hypothesis through exposure functions.
- Translates the testing problem into graphical operations on the null exposure graph.
- Future work:

  understand power properties (see power study, Slide 44);
  more hypotheses (intersection hypothesis, Slide 41)
  optimized clique decomposition.

Thank You!

Working paper: "A Graph-Theoretic Approach to Randomization Tests of Causal Effects Under General Interference"

Athey, Eckles, Imbens,"Exact p-Values for Network Interference" (JASA, 2018)

Basse, Feller, Toulis, "Randomization tests of causal effects under interference" (Biometrika, 2019)

Aronow, "A general method for detecting interference between units in randomized experiments." (Sociol. Methods Res., 2012)

Collazos, D., Garcia, E., Mejia, D., Ortega, D., and Tobon, S., "Hot spots policing in a high crime environment: An experimental evaluation in Medellin". Documento CEDE, (2019-01).

Extra slides

Experiment and data

Units and treatment assignment

- 37,055 total streets (units)
- 967 streets are identified as crime "hotspots"
- 384 are treated with increased police presence

Access to randomizations based on the design, $\mathrm{pr}(Z)$

Outcomes and covariates

- Crime counts on all streets (murders, car and motorbike thefts, personal robberies, assaults)
- Survey data on hotspot streets
- Characteristics of hotspots (distance from school, bus stop, rec center, church, neighborhood, ...)

Extensions of H_0

Athey et al (2018) consider more complex hypotheses than what can be defined based on exposures; e.g.:

$$H_0 : Y_i(z) = Y_i(z') \;\text{ if }\; z_i = z_i'.$$

This H_0 is an intersection hypothesis:

Define $f_i(z) = z_i$. Then H_0 is an intersection of:

$$H_0^0 : Y_i(z) = Y_i(z') \;\text{ if }\; f_i(z) = f_i(z') = 0. \tag{1}$$

$$H_0^1 : Y_i(z) = Y_i(z') \;\text{ if }\; f_i(z) = f_i(z') = 1. \tag{2}$$

We can still apply our method by extending the definition of the null-exposure graph:

$$\widetilde{E} = \left\{(i, z) \in \mathbb{U} \times \mathbb{Z} : f_i(z) = Z_i^{\text{obs}}\right\}. \tag{3}$$

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Definition. Let $\mathbb{U}, \mathbb{Z}$ denote the units and assignments, respectively. Let $\mathrm{a}, \mathrm{b} \in \mathcal{E}$ be any two exposures and consider the hypothesis:

$H_0^{\mathrm{a},\mathrm{b}} : Y_i(z) = Y_i(z'),$ for all i, z, z' such that $f_i(z), f_i(z') \in \{\mathrm{a}, \mathrm{b}\}$.

Define the vertex set as $V = \mathbb{U} \cup \mathbb{Z}$, and the edge set as

$$E = \{(i, z) \in \mathbb{U} \times \mathbb{Z} : f_i(z) \in \{\mathrm{a}, \mathrm{b}\}\}. \tag{4}$$

Then, $G_f = (V, E)$ is the null-exposure graph of $H_0^{\mathrm{a},\mathrm{b}}$ wrt f.

- For given $H_0^{\mathrm{a},\mathrm{b}}$ and $\{f_i\}$ the null exposure graph G_f is unique.
- Imputation is possible within the clique that contains obs. Z:

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Proposition. Consider a null-exposure graph, G_f, with some clique $C = (U, \mathcal{Z})$. If $Z^{\text{obs}} \in \mathcal{Z}$, then $Y_i(z) = Y_i(Z^{\text{obs}})$ under $H_0^{\mathrm{a,b}}$, for all $i \in U$ and all $z \in \mathcal{Z}$.

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Clique Decomposition

Let $\mathbb{U}$ be the set of units and $\mathbb{Z}$ the set of population assignments. A clique decomposition, $\mathcal{C} = \{C_1, \ldots, C_K\}$, of the null-exposure graph is a finite set of cliques, $C_k = (\mathbb{U}_k, \mathcal{Z}_k)$, $k = 1, \ldots, K$, such that

$$\bigcup_k \mathcal{Z}_k = \mathbb{Z}, \text{ and } \mathcal{Z}_k \bigcap \mathcal{Z}_{k'} = \emptyset, \text{ for any } k \neq k'.$$

$\hookrightarrow$ *The set of units does not need to be partitioned.*

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Power study: clustered interference

To illustrate, we consider a **clustered interference** setting.

Suppose we have N units spread equally in K clusters.The clusters could be classrooms or households.

Experiment: Randomly treat K/2 clusters. Within each treated cluster, randomly treat 1 unit.

$\hookrightarrow$*Motivated by student absenteeism study (Basse et al, 2019).*

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◦ Do outcomes of a control unit in control cluster differ from outcomes of a control unit in a treated cluster?

The null and competing methods

$$H_0: \quad Y_i(\texttt{control}) = Y_i(\texttt{exposed}), \ \forall i,$$

where:
$f_i(Z) = \texttt{control}$, if $Z_i = 0$ and $\sum_{j \in [i]} Z_j = 0$;
$f_i(Z) = \texttt{exposed}$, if $Z_i = 0$ and $\sum_{j \in [i]} Z_j = 1$, and $[i]$ denotes i's cluster.

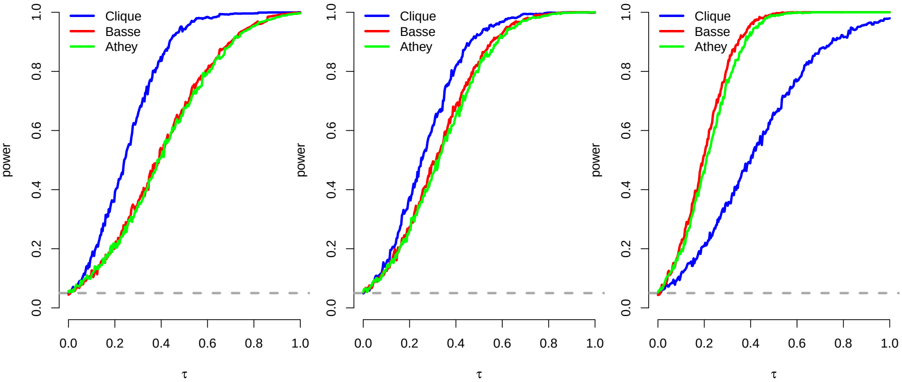
1. Athey et. al. (2018): sample one focal per household. Run randomization test*.

2. Basse et. al. (2019): For treated households, sample one untreated focal unit (uniformly). For untreated households, sample one focal. Run permutation test on the focals.

3. Clique test – proposed method.

Power comparison: $Y_i(\texttt{exposed}) = Y_i(\texttt{control}) + \tau$

$N = 300, K = 20$ $\quad$ $N = 300, K = 30$ $\quad$ $N = 300, K = 75.$

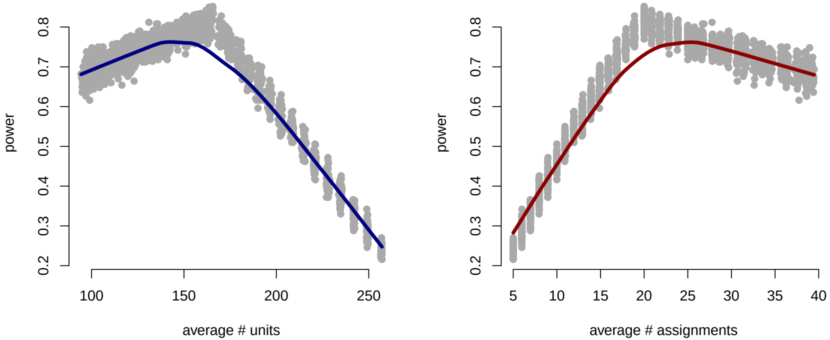


The clique test improves upon existing methods as the cluster size increases (smaller K)!

$\hookrightarrow$ *It achieves more flexible conditioning (i.e., many units/cluster).*

Power characteristics

Trade-off between #units,#assignments in the cliques.



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